



Satellite Communications

Orbits and Launching Methods

Lecture 2

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Kepler's laws



- Satellites orbit the earth following the same laws that govern the motion of the planets around the sun
- **Empirical** laws describing planetary motion
- Kepler **derived** empirically three laws describing planetary motion
- Kepler's laws apply quite generally to any two bodies in space which interact through gravitation (earth → primary, satellite → secondary)

Kepler's first law



- “the path followed by a satellite around the Earth will be an ellipse”

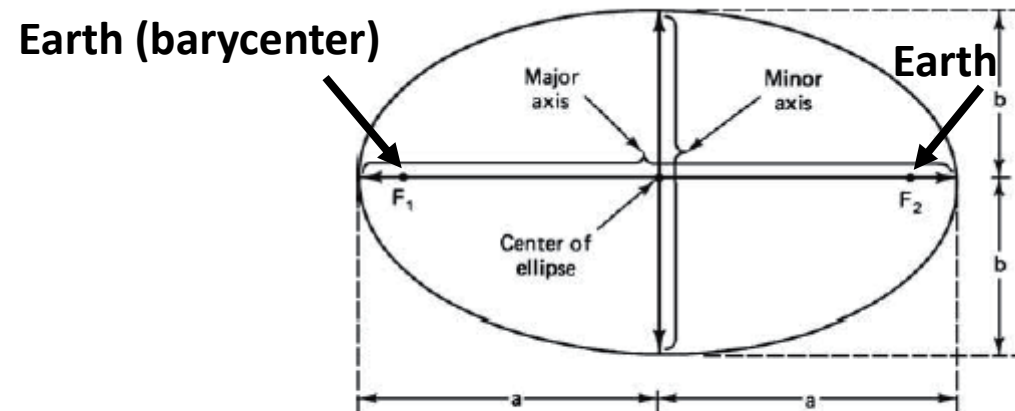


Figure 2.1 The foci F_1 and F_2 , the semimajor axis a , and the semiminor axis b of an ellipse.

- The **semi-major** and semi-minor axes of the ellipse (**a** and **b**) are related to **eccentricity e** (such that $0 < e < 1$) by

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

Kepler's second law



- “for equal time intervals, a satellite will sweep out equal areas in its orbital plane, focused at the barycenter”

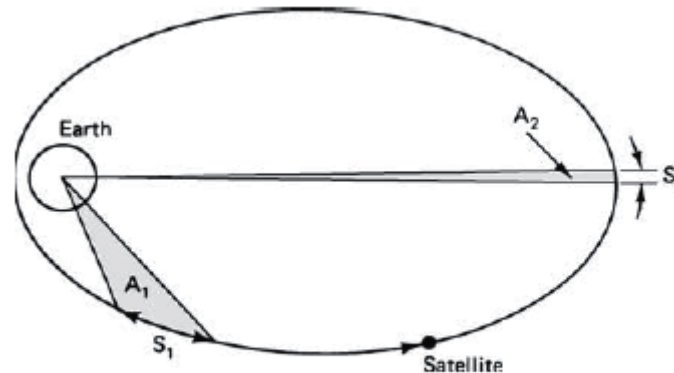


Figure 2.2 Kepler's second law. The areas A_1 and A_2 swept out in unit time are equal.

- Satellite takes longer to travel a given distance when it is farther away from earth (Explain ?) → This can be useful !



Kepler's third law

- “the square of the periodic time of orbit is proportional to the cube of the mean distance between the two bodies”

$$a^3 = \frac{\mu}{n^2}$$

- Mathematically:

- $\mu = 3.986005 \times 10^{14} \text{ m}^3/\text{s}^2$ is the earth's geocentric gravitational constant
- mean distance is equal to the **semi-major** axis **a**
- n is the mean motion of the satellite in radians per second

- Orbital period P is related to n by $P = \frac{2\pi}{n}$
- applies only to the ideal situation of a satellite orbiting a perfectly spherical earth of uniform mass, with no perturbing forces acting, such as atmospheric drag

Kepler's third law



- Ex: Calculate the radius of a circular orbit for which the period is 1 day.
- Sol:

$$n = \frac{2\pi}{24 \times 60 \times 60} = 7.272 \times 10^{-5} \text{ rad/s}$$

$$a = \left[\frac{3.986005 \times 10^{14}}{(7.272 \times 10^{-5})^2} \right]^{\frac{1}{3}} = 42,241 \text{ km}$$

Definitions of Terms for Earth-Orbiting Satellites



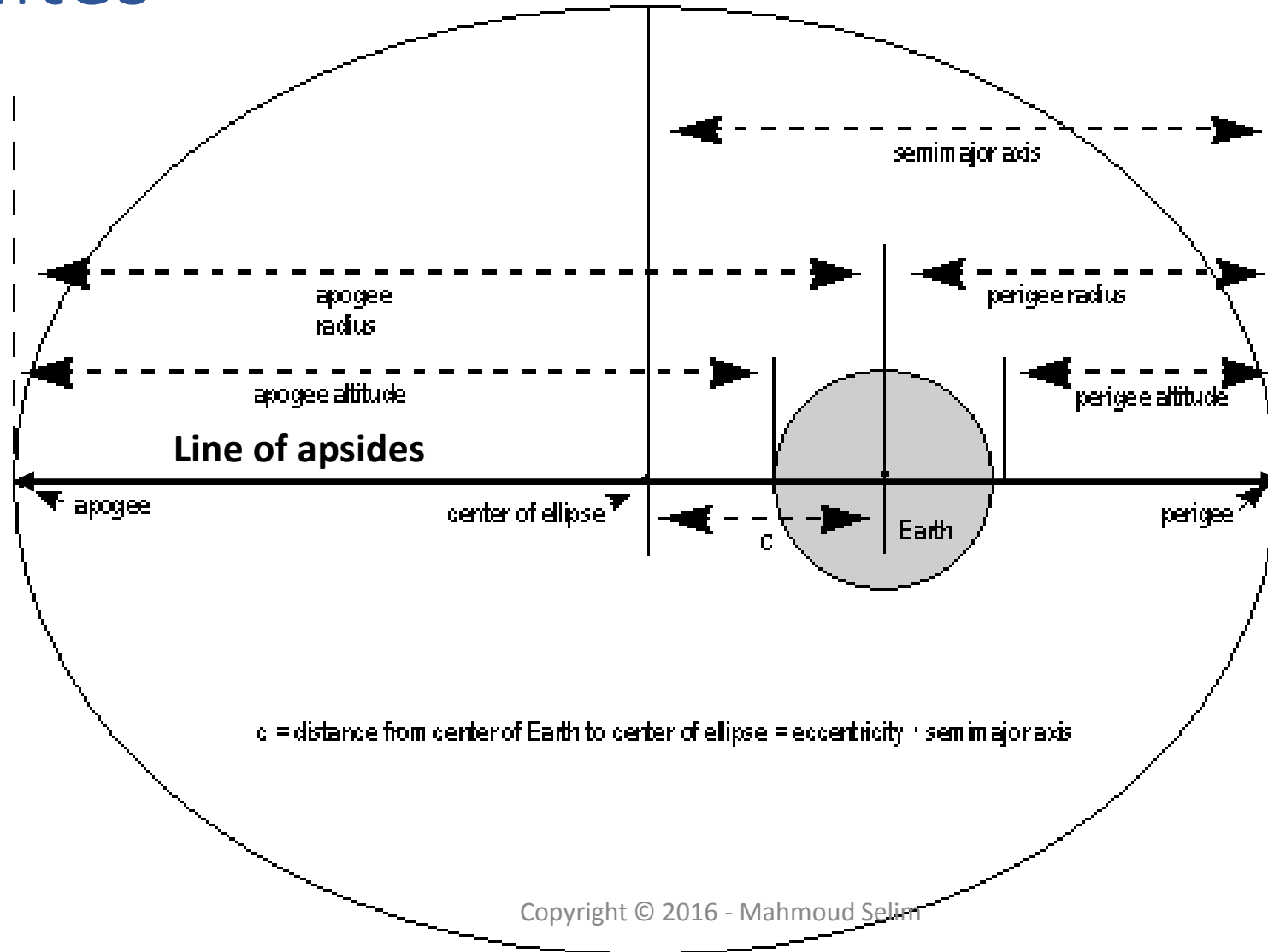
Parameter	Definition
<i>Sub-satellite path</i>	The path traced out on the earth's surface directly below the satellite
<i>Apogee</i>	The point farthest from earth (h_a)
<i>Perigee</i>	The point of closest approach to earth (h_p)
<i>Line of apsides</i>	The line joining the perigee and apogee through the center of the earth
<i>Ascending node</i>	The point where the orbit crosses the equatorial plane going from south to north
<i>Descending node</i>	The point where the orbit crosses the equatorial plane going from north to south
<i>Line of nodes</i>	The line joining the ascending and descending nodes through the center of the earth
<i>Inclination</i>	• The angle between the orbital plane and the earth's equatorial plane • measured at ascending node from the equator to the orbit, going from east to north • the greatest latitude, north or south, reached by sub-satellite path equal to inclination
<i>Prograde orbit</i>	An orbit in which the satellite moves in the same direction as the earth's rotation ($0 \rightarrow 90$)/ (<i>direct orbit</i>)/ save launch energy

Definitions of Terms for Earth-Orbiting Satellites



Parameter	Definition
<i>Retrograde orbit</i>	An orbit in which the satellite moves in a direction counter to the earth's rotation (90 → 180)
<i>Argument of perigee</i> ω	The angle from ascending node to perigee, measured in the orbital plane at the earth's center, in the direction of satellite motion
<i>Right ascension of the ascending node</i> Ω	- Longitude of the ascending node is not fixed because the earth spins (Not a reference !) - Longitude and time are both required, but fixed reference in space is required also ! - Equinox (Vernal) → when the sun crosses the equator going from south to north, and an imaginary line (line of aries) drawn from this equatorial crossing through the center of the sun points to the Equinox - The right ascension of the ascending node is then the angle measured eastward, in the equatorial plane, from this line to the ascending node
<i>Mean anomaly</i>	an average value of the angular position of the satellite with reference to the perigee
<i>True anomaly</i>	the angle from perigee to the satellite position, measured at the earth's center

Definitions of Terms for Earth-Orbiting Satellites



Definitions of Terms for Earth-Orbiting Satellites

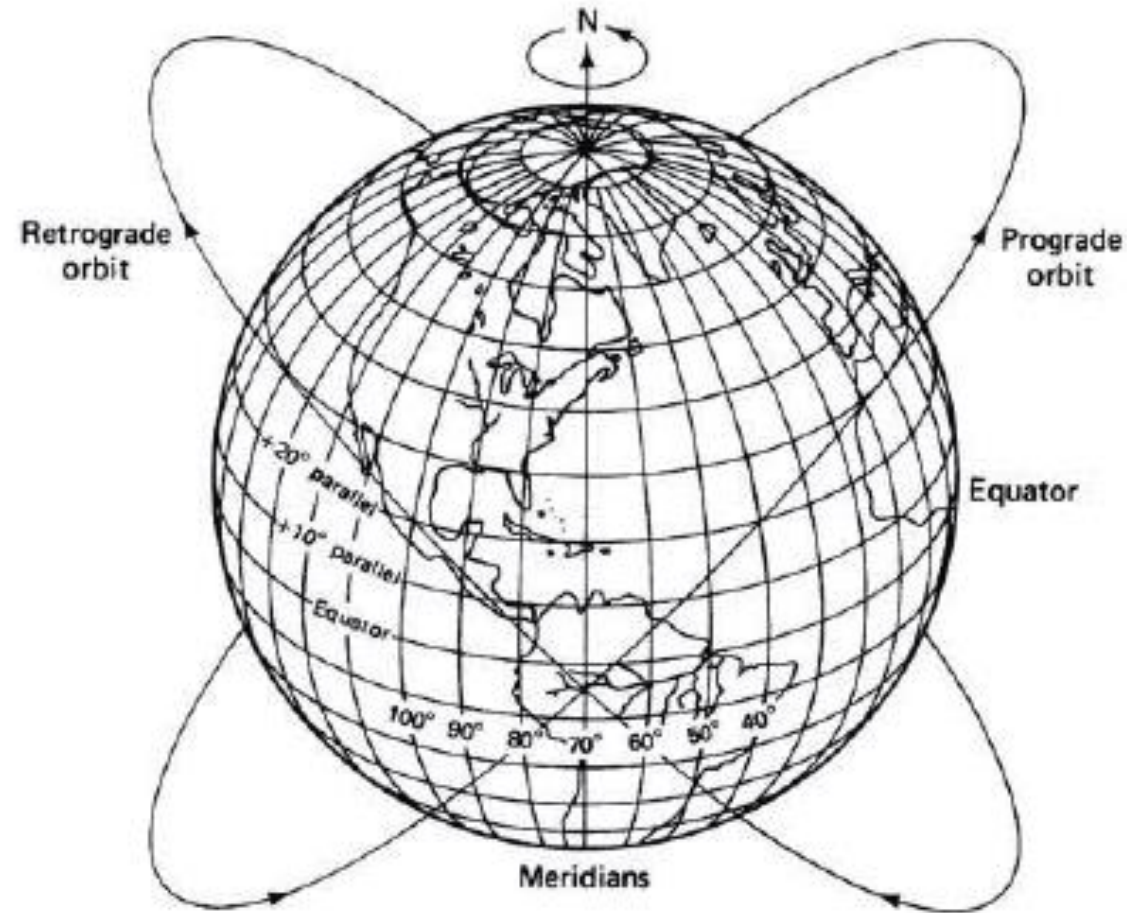
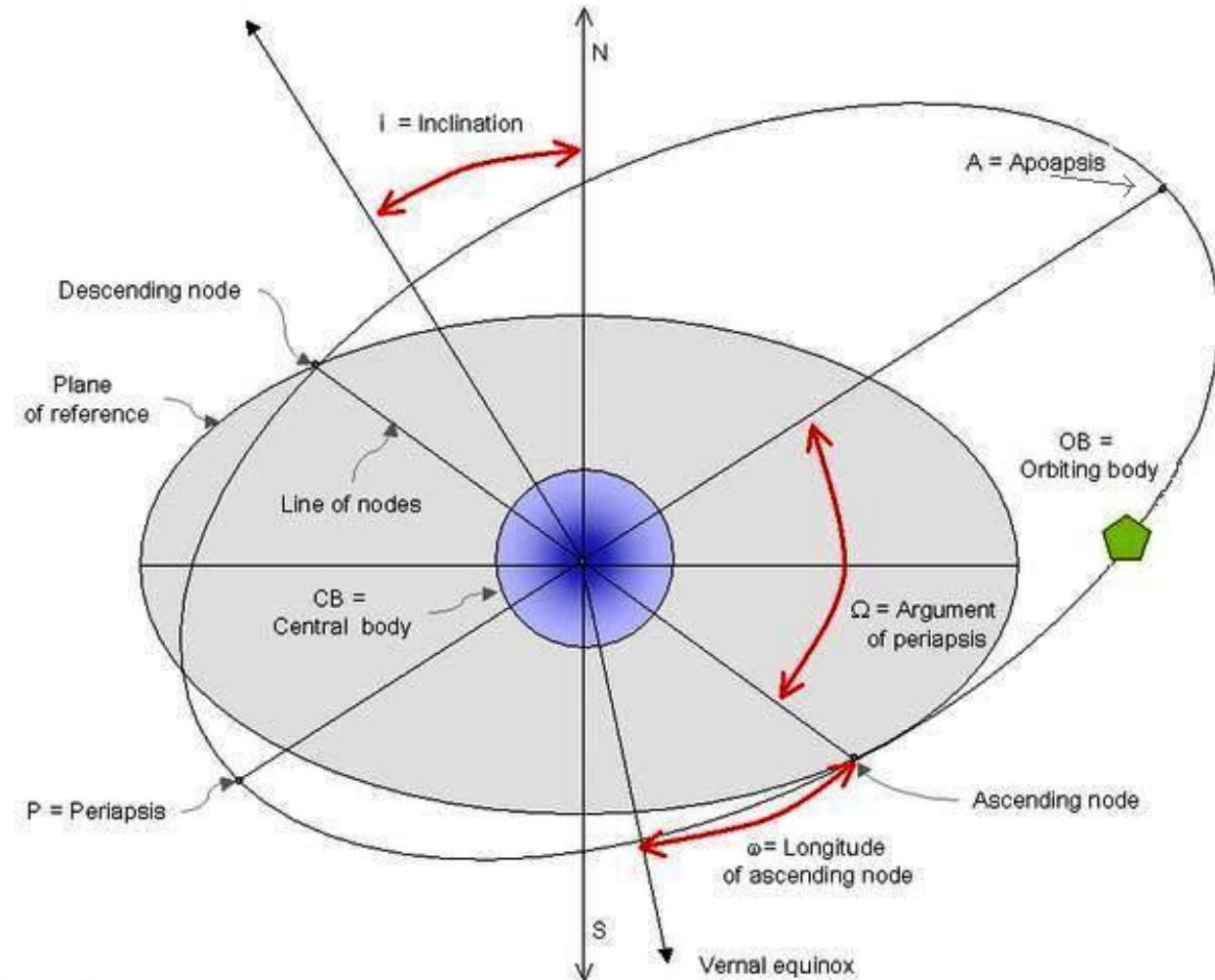
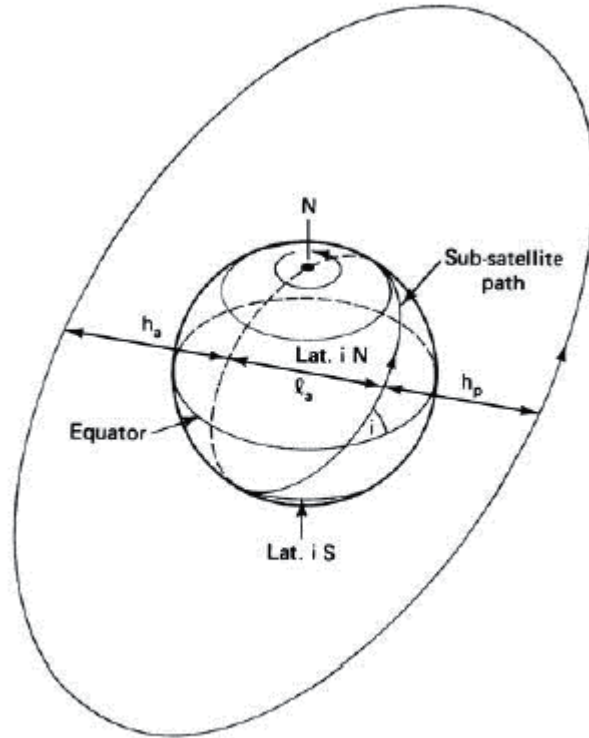


Figure 2.4 Prograde and retrograde orbits.

Definitions of Terms for Earth-Orbiting Satellites



Definitions of Terms for Earth-Orbiting Satellites

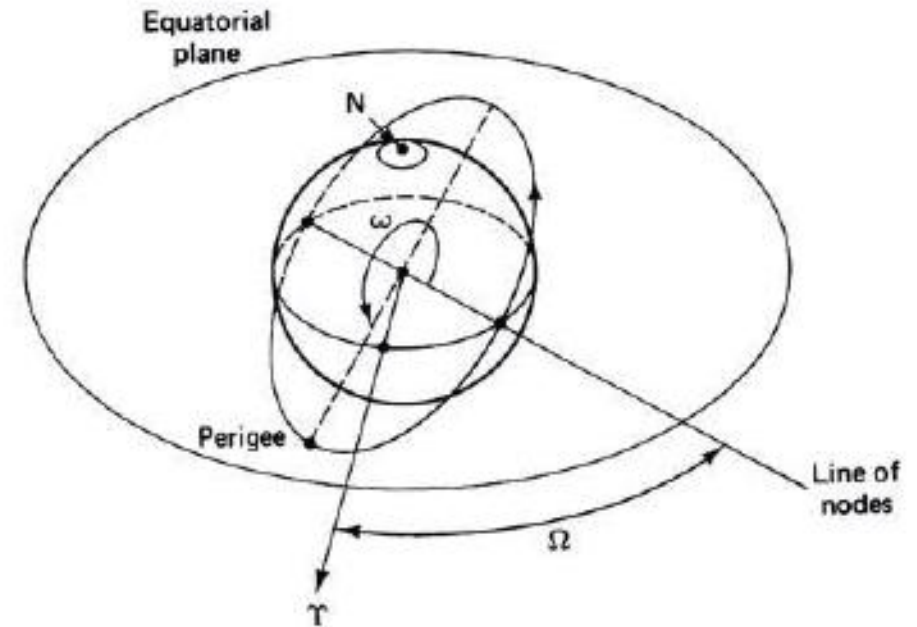
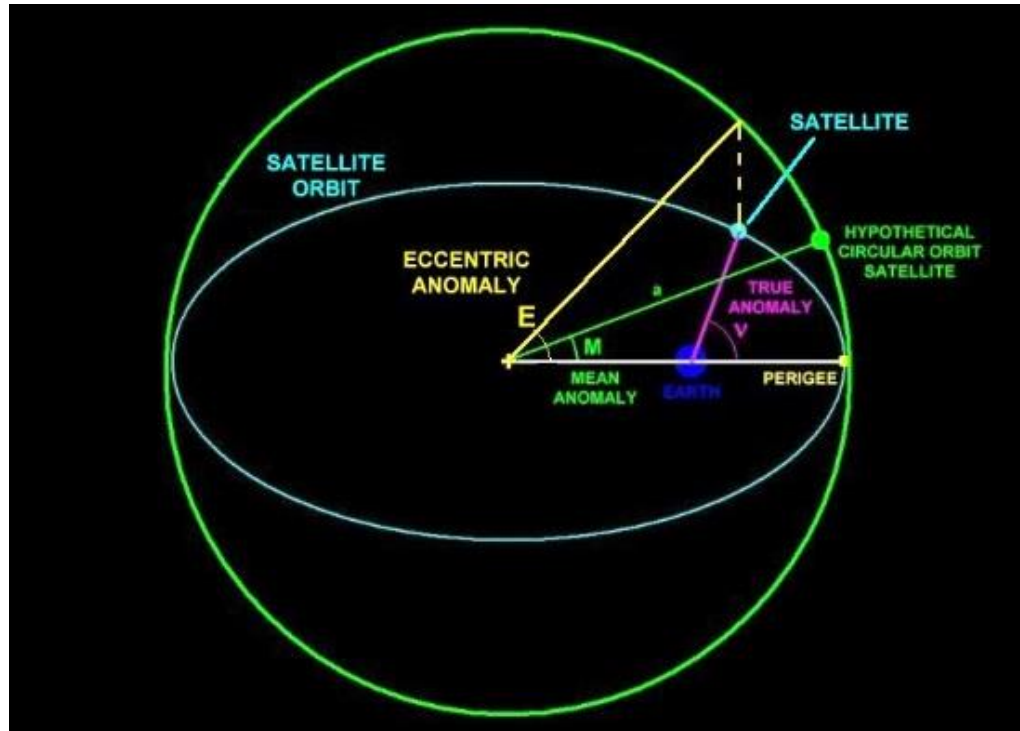


Figure 2.5 The argument of perigee ω and the right ascension of the ascending node Ω .

Orbital Elements



- Earth-orbiting artificial satellites are defined by six orbital elements referred to as the ***keplerian element set***:
 - **semimajor axis a** and **eccentricity e** → determines the shape of the orbit
 - **mean anomaly M_0** → gives the position of the satellite in its orbit at a reference time (epoch)
 - **Argument of perigee** → gives the rotation of the orbit's perigee point relative to the orbit's line of nodes in the earth's equatorial plane
 - **the inclination** and the **right ascension of the ascending node** → relate the orbital plane's position to the earth

Rev. No.

Orbital Elements



- <http://celestrak.com/NORAD/elements/>
- Ex: from the following table, Calculate the semi-major axis for the satellite
- Sol:

Mean motion is given in rev/day as

$$NN = 14.23304826 \text{ day}^{-1}$$

$$n_0 = 2 \times \pi \times NN = 0.00104 \text{ s}^{-1}$$

$$a = \left[\frac{\mu}{n_0^2} \right]^{1/3} = 7192.335 \text{ km}$$

TABLE 2.1 Details from the NASA Bulletins (see Fig. 2.6 and App. C)

Line no.	Columns	Description
1	3-7	<i>Satellite number: 25338</i>
1	19-20	<i>Epoch year (last two digits of the year): 00</i>
1	21-32	<i>Epoch day (day and fractional day of the year): 223.79688452 (this is discussed further in Sec. 2.9.2)</i>
1	34-43	<i>First time derivative of the mean motion (rev/day³): 0.00000307</i>
2	9-16	<i>Inclination (degrees): 98.6328</i>
2	18-25	<i>Right ascension of the ascending node (degrees): 251.5324</i>
2	27-33	<i>Eccentricity (leading decimal point assumed): 0011501</i>
2	35-42	<i>Argument of perigee (degrees): 113.5534</i>
2	44-51	<i>Mean anomaly (degrees): 246.6853</i>
2	53-63	<i>Mean motion (rev/day): 14.23304826</i>
2	64-68	<i>Revolution number at epoch (rev): 11,663</i>

Apogee and Perigee Heights



- From the ellipse geometry, apogee and perigee radii are given as (derive !)
 - Apogee radius: $r_a = a(1 + e)$ and Perigee radius: $r_p = a(1 - e)$
 - Apogee and Perigee height are obtained by subtraction from earth radius
- Ex: Calculate the apogee and perigee heights for the previous satellite parameters assuming mean earth radius is $R = 6371 \text{ km}$
- Sol: from the table $e = 0.0011501$ and from the previous ex. $a = 7192.33 \text{ km}$
$$r_a = a(1 + e) = 7192.33(1 + 0.0011501) = 7200.607 \text{ km}$$
$$r_p = a(1 - e) = 7192.33(1 - 0.0011501) = 7184.063 \text{ km}$$
and the corresponding heights are
$$h_a = r_a - R = 7200.607 - 6371 = 829.6 \text{ km}$$
$$h_p = r_p - R = 7184.063 - 6371 = 813.1 \text{ km}$$

Orbit Perturbations



- Previous discussion assumes: uniform spherical earth, equilibrium of centrifugal force (satellite) and gravitational pull (earth) → **IDEAL** !
- Non-spherical earth (*oblate spheroid*) and atmospheric drag should be considered

1. Non-spherical earth: Equatorial bulge and Poles flattening

- The real mean motion n is modified to (in terms of n_0 , a , e and i):

$$n = n_0 \left[1 + \frac{K_1 (1 - 1.5 \sin^2 i)}{a^2 (1 - e^2)^{1.5}} \right] \quad K_1 = 66,063.1704 \text{ Km}^2$$

- The orbital period (anomalistic period) is $P_A = \frac{2\pi}{n} \text{ s}$
- The mean motion is usually specified in the satellite parameters or obtained as the reciprocal of the anomalistic period in rad/s
- Earth oblateness has negligible effect on semimajor axis a

Orbit Perturbations



- Ex: A satellite is orbiting in the equatorial plane with a period from perigee to perigee of 12 h. Given that the eccentricity is 0.002, calculate the semi-major axis. The earth's equatorial radius is 6378.1414 km

Solution Given data: $e = 0.002$; $i = 0^\circ$; $P = 12$ h; $K_1 = 66063.1704 \text{ km}^2$; $a_E = 6378.1414 \text{ km}$; $\mu = 3.986005 \times 10^{14} \text{ m}^3/\text{s}^2$

The mean motion is:

$$n = \frac{2\pi}{P} \\ = 1.454 \times 10^{-4} \text{ s}^{-1}$$

• Sol:

Assuming this is the same as n_0 , Kepler's third law gives

$$a = \left(\frac{\mu}{n^2} \right)^{1/3} \\ = \underline{\underline{26610 \text{ km}}}$$

Solving the root equation

$$n - \sqrt{\frac{\mu}{a^3}} \left[1 + \frac{K_1(1 - 1.5 \sin^2 i)}{a^2(1 - e^2)^{1.5}} \right] = 0$$

yields a value of 26,612 km



Orbit Perturbations



- Regression of the nodes (due to oblateness of the earth)
 - the nodes appear to slide along the equator resulting is rotation of line of nodes and shift in the right ascension of the ascending node
 - If orbit is prograde → nodes slide westward and if the orbit is retrograde → nodes slide eastward (hence regression !)
- rotation of apsides in the orbital plane
- The mean motion **n**, the semimajor axis **a** and eccentricity **e** determines the amount of both effects through a factor K such that

$$K = \frac{nK_1}{a^2(1-e^2)^2} \quad \textbf{(rad/day)}$$

- The rate of change of Ω with respect to time is $\frac{d\Omega}{dt} = -K \cos i$
- The rate of change of ω with respect to time is $\frac{d\omega}{dt} = K(2 - 2.5 \sin^2 i)$

Orbit Perturbations



- At any time t , with respect to the epoch time t_0 , the angles are:

$$\omega = \omega_0 + \frac{d\omega}{dt}(t - t_0) \qquad \Omega = \Omega_0 + \frac{d\Omega}{dt}(t - t_0)$$

- Thus, satellite drifts from elliptical path due to both effect (orbit itself is moving due to change in ω and Ω)

Orbit Perturbations



- **Ex:** Determine the rate of regression of the nodes and the rate of rotation of the line of apsides for the satellite parameters specified in Table 2.1. The value for a obtained in previous Example may be used.

Orbit Perturbations



Solution From Table 2.1 and Example 2.2 $i = 98.6328^\circ$; $e = 0.0011501$; $NN = 14.23304826 \text{ day}^{-1}$; $a = 7192.335 \text{ km}$, and the known constant: $K_1 = 66063.1704 \text{ km}^2$

Converting n to rad/s:

$$n = 2\pi NN$$

From Eq. (2.11):

$$\begin{aligned} K &= \frac{nK_1}{a^2(1 - e^2)^2} \\ &= 6.544 \text{ deg/day} \end{aligned}$$

From Eq. (2.12):

$$\begin{aligned} \frac{d\Omega}{dt} &= -K \cos i \\ &= 0.981 \text{ deg/day} \end{aligned}$$

From Eq. (2.13):

$$\begin{aligned} \frac{d\omega}{dt} &= K(2 - 2.5 \sin^2 i) \\ &= \underline{\underline{-2.904 \text{ deg/day}}} \end{aligned}$$

Orbit Perturbations



- **Ex:** Calculate, for the satellite in previous example, the new values for ω and Ω one period after epoch (use the table)

Orbit Perturbations



Solution From Table 2.1:

$$NN = 14.23304826 \text{ day}^{-1}; \omega_0 = 113.5534^\circ; \Omega_0 = 251.5324^\circ$$

The anomalistic period is

$$\begin{aligned} P_A &= \frac{1}{NN} \\ &= 0.070259 \text{ day} \end{aligned}$$

This is also the time difference $(t - t_0)$ since the satellite has completed one revolution from perigee to perigee. Hence:

$$\begin{aligned} \Omega &= \Omega_0 + \frac{d\Omega}{dt}(t - t_0) \\ &= 251.5324 + 0.981(0.070259) \\ &= \underline{\underline{251.601^\circ}} \end{aligned}$$

$$\begin{aligned} \omega &= \omega_0 + \frac{d\omega}{dt}(t - t_0) \\ &= 113.5534 + (-2.903)(0.070259) \\ &= \underline{\underline{113.349^\circ}} \end{aligned}$$

Orbit Perturbations



- Equatorial ellipticity (of the order of 10^{-5}) (because earth is not perfectly circular in equatorial plane) results in gravity gradient that affects satellites in geostationary orbits to drift rather than being fixed to earth
- This effect is handled by station-keeping maneuvers

Orbit Perturbations



2. Atmospheric drag

- Affects near earth satellites (below about 1000 km)
- The effect is highest at perigee → reduction in velocity → apogee height decrease successively → **e** and **a** decrease
- Significant for long time intervals
- Approximate expression for majoraxis change at time t

$$a \cong a_0 \left[\frac{n_0}{n_0 + n'_0(t - t_0)} \right]^{2/3}$$

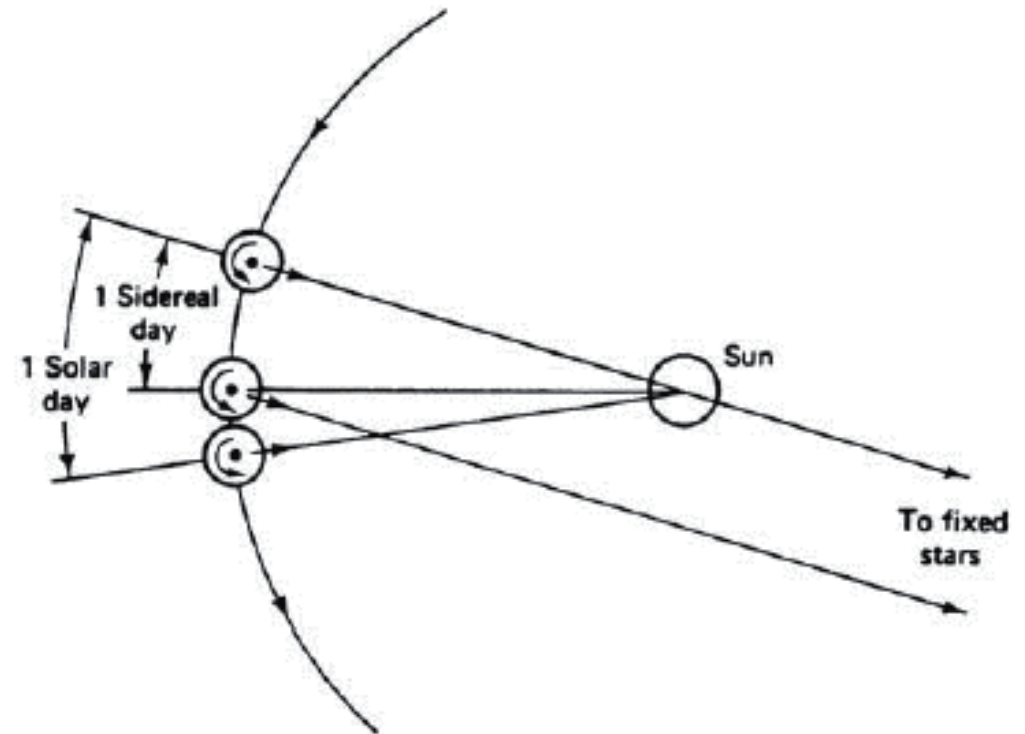
- The change in mean anomaly is

$$\delta M = \frac{n'_0}{2} (t - t_0)^2$$

Calendars



- Tropical year contains 365.2422 mean solar days (relative to mean sun) is divided into 365 days
- The extra 0.2422 day is corrected by the “leap year” → Julian Ceaser → Julian Calendar
- Gregorian Calendar is in use today → Pope Gregory XIII by solving a problem by missing out 3 days every 400 years
- Sidereal day: one complete rotation of the earth relative to the fixed stars (1 mean solar day = 1.0027379093 mean sidereal day)



Universal time



- UTC → Universal Time Coordinated
- Reference time for setting clocks
- The fundamental unit is the mean solar day (24h,60m,60s)
- Epoch time is given in terms of UTC
- Similar to Greenwich mean time (GMT) and Zulu (Z) time
- UT is expressed in two forms: as a fraction of a day and in degrees

$$UT_{day} = \frac{1}{24} \left[hours + \frac{minutes}{60} + \frac{seconds}{3600} \right]$$

$$UT^{\circ} = 360^{\circ} \times UT_{day}$$

The orbital plane



- From geometry, magnitude of vector r is

$$r = \frac{a(1-e^2)}{1+e\cos\nu}$$

- How to find ν (true anomaly) which is varying with time ?!
- **First**, find the mean anomaly M at time t as

$$M = n(t - T_p)$$

- But, $T_p = t_0 - \frac{M_0}{n}$
- Thus, $M = M_0 + n(t - t_0)$

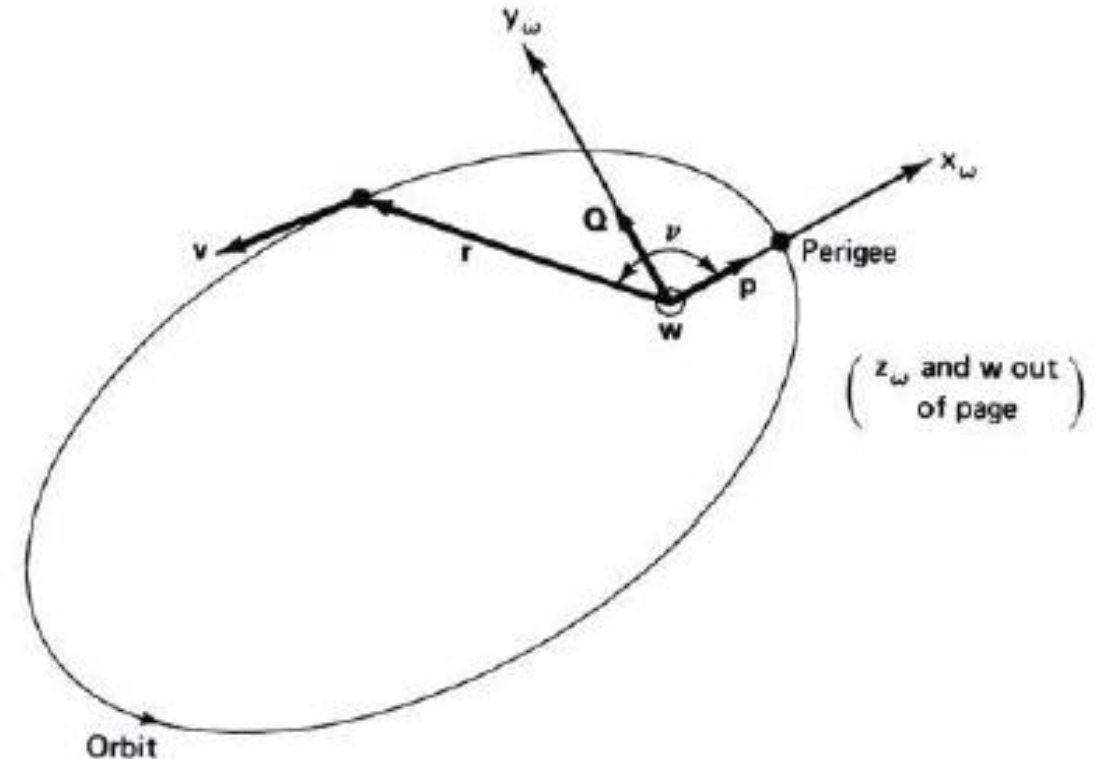


Figure 2.8 Perifocal coordinate system (PQW frame).

The orbital plane



- **Secondly,** solve Kepler's equation for an intermediate variable E (Eccentric anomaly) as
$$M - (E - e \sin E) = 0$$

- Solved using iterative method !
- Find ν using Gauss's equation:

$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

- The magnitude of vector r can also be obtained from E as

$$r = a(1 - e \cos E)$$

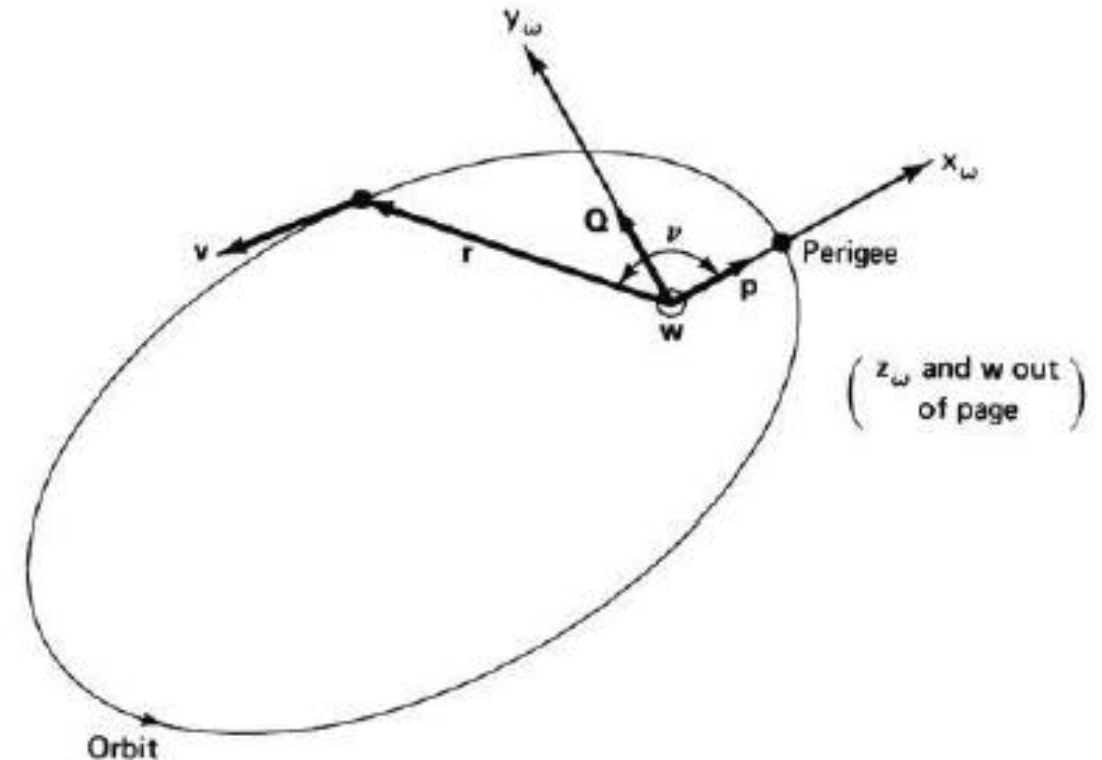


Figure 2.8 Perifocal coordinate system (PQW frame).

The orbital plane



- For near-circular orbits, ν can be obtained directly from M as

$$\nu \cong M + 2e \sin M + \frac{5}{4}e^2 \sin 2M$$

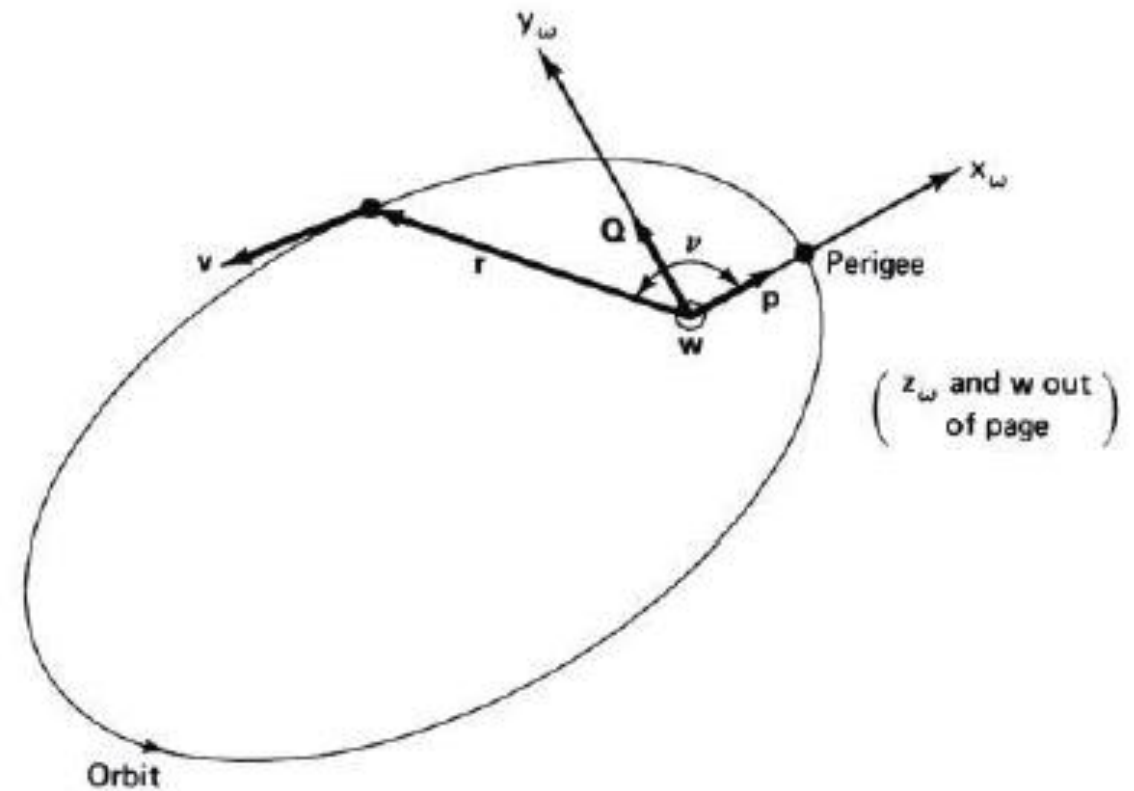


Figure 2.8 Perifocal coordinate system (PQW frame).

The orbital plane



- **Ex:** For a given satellite, $e = 9.5981 \times 10^{-3}$ and mean anomaly is 204.9779 deg., the mean motion is 14.2171404 rev/day, $a = 7194.9$ Km

Calculate the true anomaly, the magnitude of the radius vector 5 s after epoch

The orbital plane



Solution The rotation in radians per second is

$$n = \frac{14.2171404 \times 2\pi}{86400} \\ \cong 0.001034 \text{ rad/s}$$

• **Sol:**

The mean anomaly of 204.9779° , in radians is 3.57754, and 5 s after epoch the mean anomaly becomes

$$M = 3.57754 + 0.001034 \times 5 \\ \cong 3.5827 \text{ rad} \\ \nu \cong 3.5827 + 2 \times 9.5981 \times 10^{-3} \times \sin 3.5827 \\ + \frac{5}{4} \times (9.5981 \times 10^{-3})^2 \times \sin(2 \times 3.5827) \\ = \underline{\underline{3.5746 \text{ rad}}} \quad (\underline{\underline{= 204.81^\circ}})$$

Applying Eq. (2.23) gives r as

$$r = \frac{7194.9 \times (1 - 9.5981^2) \times 10^{-6}}{1 + 9.5981 \times 10^{-3} \times \cos 204.81} \\ = \underline{\underline{7257.5 \text{ km}}}$$